5. Two particles move along the x-axis. For $0 \leq t \leq 8$, the position of particle $P$ at time $t$ is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle $Q$ at time $t$ is given by $v_Q(t) = t^2 - 8t + 15$.

Particle $Q$ is at position $x = 5$ at time $t = 0$.

(a) For $0 \leq t \leq 8$, when is particle $P$ moving to the left?

(b) For $0 \leq t \leq 8$, find all times $t$ during which the two particles travel in the same direction.

(c) Find the acceleration of particle $Q$ at time $t = 2$. Is the speed of particle $Q$ increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

(d) Find the position of particle $Q$ the first time it changes direction.
Question 5

(a) \[ x'_p(t) = \frac{2t - 2}{t^2 - 2t + 10} = \frac{2(t - 1)}{t^2 - 2t + 10} \]

\[ t^2 - 2t + 10 > 0 \text{ for all } t. \]

\[ x'_p(t) = 0 \Rightarrow t = 1 \]
\[ x'_p(t) < 0 \text{ for } 0 \leq t < 1. \]

Therefore, the particle is moving to the left for \( 0 \leq t < 1. \)

(b) \[ v_Q(t) = (t - 5)(t - 3) \]

\[ v_Q(t) = 0 \Rightarrow t = 3, t = 5 \]

\[ \begin{array}{cccccccc}
0 & 1 & + & v_p(t) & 8 & t \\
\end{array} \]

\[ \begin{array}{cccccccc}
0 & 3 & 5 & 8 & v_Q(t) \\
+ & 0 & - & 0 & + \\
\end{array} \]

Both particles move in the same direction for \( 1 < t < 3 \) and \( 5 < t \leq 8 \) since \( v_p(t) = x'_p(t) \) and \( v_Q(t) \) have the same sign on these intervals.

(c) \[ a_Q(t) = v'_Q(t) = 2t - 8 \]

\[ a_Q(2) = 2 \cdot 2 - 8 = -4 \]

\[ a_Q(2) < 0 \text{ and } v_Q(2) = 3 > 0 \]

At time \( t = 2 \), the speed of the particle is decreasing because velocity and acceleration have opposite signs.

(d) Particle \( Q \) first changes direction at time \( t = 3. \)

\[ x_Q(3) = x_Q(0) + \int_0^3 v_Q(t) \, dt = 5 + \int_0^3 (t^2 - 8t + 15) \, dt \\
= 5 + \left[ \frac{1}{3} t^3 - 4t^2 + 15t \right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23 \]
Question 2

For \( t \geq 0 \), a particle moves along the \( x \)-axis. The velocity of the particle at time \( t \) is given by
\[
v(t) = 1 + 2 \sin \left( \frac{t^2}{2} \right).
\]
The particle is at position \( x = 2 \) at time \( t = 4 \).

(a) At time \( t = 4 \), is the particle speeding up or slowing down?

(b) Find all times \( t \) in the interval \( 0 < t < 3 \) when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time \( t = 0 \).

(d) Find the total distance the particle travels from time \( t = 0 \) to time \( t = 3 \).
A particle moves along a straight line. For $0 \leq t \leq 5$, the velocity of the particle is given by
\[ v(t) = -2 + \left( t^2 + 3t \right)^{6/5} - t^3, \]
and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of $t$ in the interval $2 \leq t \leq 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times $t$ in the interval $0 \leq t \leq 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.
For $0 \leq t \leq 12$, a particle moves along the $x$-axis. The velocity of the particle at time $t$ is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \leq t \leq 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time $t$. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$. 
For \( 0 \leq t \leq 6 \), a particle is moving along the x-axis. The particle’s position, \( x(t) \), is not explicitly given. The velocity of the particle is given by \( v(t) = 2 \sin\left(e^{t/4}\right) + 1 \). The acceleration of the particle is given by \( a(t) = \frac{1}{2}e^{t/4} \cos\left(e^{t/4}\right) \) and \( x(0) = 2 \).

(a) Is the speed of the particle increasing or decreasing at time \( t = 5.5 \)? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period \( 0 \leq t \leq 6 \).

(c) Find the total distance traveled by the particle from time \( t = 0 \) to \( t = 6 \).

(d) For \( 0 \leq t \leq 6 \), the particle changes direction exactly once. Find the position of the particle at that time.
Two particles move along the x-axis. For $0 \leq t \leq 6$, the position of particle $P$ at time $t$ is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle $R$ at time $t$ is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \leq t \leq 6$, find all times $t$ during which particle $R$ is moving to the right.

(b) For $0 \leq t \leq 6$, find all times $t$ during which the two particles travel in opposite directions.

(c) Find the acceleration of particle $P$ at time $t = 3$. Is particle $P$ speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \leq t \leq 3$. 
A particle moves along the $x$-axis with position at time $t$ given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$.

(a) Find the time $t$ at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant $A$ for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$. 
Question 2

A particle moves along the $x$-axis so that its velocity $v$ at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of $v$ is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time $t$ is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

(a) Find the acceleration of the particle at time $t = 3$.
(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.
(c) Find the position of the particle at time $t = 3$.
(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time $t$ at which the particle is farthest to the right. Explain your answer.
Question 3

A particle moves along the x-axis so that its velocity \( v \) at time \( t \), for \( 0 \leq t \leq 5 \), is given by 
\[ v(t) = \ln(t^2 - 3t + 3). \] 
The particle is at position \( x = 8 \) at time \( t = 0 \).

(a) Find the acceleration of the particle at time \( t = 4 \).

(b) Find all times \( t \) in the open interval \( 0 < t < 5 \) at which the particle changes direction. During which time intervals, for \( 0 \leq t \leq 5 \), does the particle travel to the left?

(c) Find the position of the particle at time \( t = 2 \).

(d) Find the average speed of the particle over the interval \( 0 \leq t \leq 2 \).
Question 3

A particle moves along the $y$-axis so that its velocity $v$ at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$.

At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.
A particle moves along the $x$-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of $t$ at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$. 

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**Question 4**

A particle moves along the $x$-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of $t$ at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$. 

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A particle moves along the $x$-axis so that its velocity at time $t$ is given by

$$v(t) = -(t + 1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times $t$ in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \leq t \leq 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.
Question 3

A particle moves along the $x$-axis so that its velocity $v$ at any time $t$, for $0 \leq t \leq 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

(a) On the axes provided, sketch the graph of $v(t)$ for $0 \leq t \leq 16$.
(b) During what intervals of time is the particle moving to the left? Give a reason for your answer.
(c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.
(d) Is there any time $t$, $0 < t \leq 16$, at which the particle returns to the origin? Justify your answer.

(a) $v(t)$

\begin{center}
\begin{tikzpicture}
\begin{axis}[
axis lines=middle,
xtick={0,4,8,12},
xticklabels={0,4,8,12},
ytick={-8,-4,0,4,8},
yticklabels={-8,-4,0,4,8},
]
\end{axis}
\end{tikzpicture}
\end{center}
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Question 3

An object moves along the x-axis with initial position \( x(0) = 2 \). The velocity of the object at time \( t \geq 0 \) is given by \( v(t) = \sin\left(\frac{\pi}{3} t\right) \).

(a) What is the acceleration of the object at time \( t = 4 \) ?

(b) Consider the following two statements.
   
   Statement I: For \( 3 < t < 4.5 \), the velocity of the object is decreasing.
   
   Statement II: For \( 3 < t < 4.5 \), the speed of the object is increasing.

   Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval \( 0 \leq t \leq 4 \)?

(d) What is the position of the object at time \( t = 4 \) ?